Mortgage Choice - The Danish Case*

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Abstract

In this paper we analyze the mortgage choice faced by Danish borrowers. Based on an analysis of the most popular Danish mortgage products, we argue that Adjustable-Rate Mortgages (ARM) with life time caps will combine the most attractive features from straight ARMs and callable Fixed-Rate Mortgages (FRM). Furthermore, we find the delivery option embedded in Danish mortgages to be an important feature that facilitates a tight match between assets and liabilities in household portfolios.

1 Introduction

The Danish market for mortgage backed bonds is more than 200 years old, but the basic principles have remained the same[1]. It is characterized by its relative simplicity and a high degree of efficiency. The so-called mortgage credit institutions fund loans issued to borrowers by selling an equal amount of bonds in the markets. A strict balance principle in the legislation requires the mortgage credit institutions to have a very close match between the payments on the loans and the bonds issued, which basically means that all issues are pass-throughs. Furthermore, independent of the borrowers creditworthiness the maximum loan-to-value LTV limits are 80% for residential property and 60% for corporate property. In effect Danish mortgage backed bonds are considered highly-secure investment grade securities with ratings from Moody's ranging from Aaa to Aa2. In fact, in the 200-year history not a single investor has received less than the full payment and not a single mortgage credit institution has gone bankrupt.

Over the years changes in the mortgage legislation as well as tax-laws have also affected the maturity and amortization profiles of the loans issued. The vast majority of the mortgages are still callable annuity bonds with maturities

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[1]More detailed information can e.g. be found in the *Mortgage Financing in Denmark* (1999).

of 30-, 20- or 10-years. However, several loan types have been, and still are available e.g. Inflation-indexed Fixed-Rate Mortgages (IFRM).

In 1989 an amendment to the existing mortgage law allowed new mortgage credit institutions to be created, and an effect of this has been a significant increase in competition. The close connection between bonds and loan profiles makes bonds from different mortgage credit institutions very close substitutes, which ultimately forces mortgage credit institutions to make costs and markups the primary parameters in a very competitive market. However, an increasingly important parameter has been the development of new mortgage products in order to maintain (and attract) customers. In our opinion the increased competition and the focus on market shares have lead the mortgage credit institutions to develop mortgage products that appear attractive at prevailing market conditions, focusing primarily on low initial payments.

The mortgage choice is the single most important financial decision most households are going to make. As discussed in e.g. Campbell & Cocco (2002) this decision requires considerations that are on the frontier of finance research, including uncertainty in interest rates and inflation, risky labor income, borrowing constraints and illiquid assets.

In the US mortgage choice literature the mobility of the borrower is a key issue. A consistent finding is that mobile and wealthy borrowers as well as short term housing tenures tend to prefer short-term mortgages (see e.g. Dhillon, Schilling & Sirmans (1987), Brueckner & Follain (1988), Aadu & Sirmans (1995)). A closely related part of the US literature argues that points and coupons are ways to make borrowers reveal private information regarding their mobility (see e.g. Leroy (1996), Stanton & Wallace (1998)).

In a very recent study Campbell & Cocco (2002) implements a life-cycle model with interest rate and income risk. Furthermore, the model includes mobility for non-financial reasons, a variety of mortgage contracts including second mortgages, and the wealth effect of the property value. Their numerical results support the findings that low probability of moving, large houses and high risk aversion increase the preferences for FRMs. Their main conclusion is that inflation-indexed FRMs are preferred for household risk management, however this effect is decreasing in inflation uncertainty. Also interestingly, they find that hybrid ARMs with caps and floors are more attractive than both straight ARMs and nominal FRMs.

However, there is particularly one reason as to why we cannot apply the conclusions from the US case in Denmark. Danish mortgage backed bonds have an additional option embedded, often denoted the delivery option (or buy back option). This option allows a mortgagor to cancel his loan by buying back bonds at market value, effectively cashing in the capital gain. This means that no prepayments are observed whenever prices are below par, but most importantly the delivery option alleviates the mobility issue known from the US and hence makes most US mortgage choice studies less applicable in the Danish case.

As we shall see, the delivery option sustains a better match between the mortgagor's assets and liabilities, and therefore we often hear the opposite advice in Denmark; borrowers that are more likely to move should issue FRMs close to par. The reason is, if they are likely to move, they should be willing to pay the higher coupon in a FRM for a short while, in return for protection against increasing interest rates just prior to a house sale.

The literature on mortgage choice in Denmark is rather limited considering

the long history and no single study has been taking all the elements considered in Campbell & Cocco (2002) into consideration. Recent research in Nielsen & Poulsen (2002*a*) and Nielsen & Poulsen (2002*b*) gives a partial explanation. Focusing on the interest rate risk using an advanced two-factor stochastic programming approach they are able to support the prepayment behavior historically observed in Denmark, including both prepayments and deliveries. However, they do not consider early redemption for non-financial reasons (death, divorce, job relocation), and therefore favor straight ARMs to FRMs in their implementation, as the delivery option is less worth in a straight ARM due to the low interest rate sensitivity. Finally, they do not consider ARMs with caps.

Indexed linked FRMs have been available in Denmark for decades, but have gradually become less popular. This is interesting considering the conclusions made by Campbell & Cocco (2002), but could well be related to the relative low inflation uncertainty in the Euro area due to the ECB policy.

In this paper we examine a hybrid ARM, Bolig-X (BLX), recently introduced in the Danish Mortgage market and compare it to existing mortgage types. Furthermore, we suggest modifications that in our opinion would make it even more attractive. Even though we do not apply a formal utility based analysis, our conclusion is not far from the one in Campbell & Cocco (2002). Given that mortgagors are risk averse it must be important to ensure a reasonable match between assets and liabilities in the household portfolio. An main point in this paper is that the Bolig-X loans with cap[2] are surprisingly reminiscent of callable FRMs. Both loan types protect the mortgagor from increases in interest rates and the mortgage payments can be reduced when interest rates decline. The existing Bolig-X loans are, however, too short to provide the mortgagor with a sufficient protection against increases in interest rates, but if the maturity of the loan and hence the cap is increased to e.g. 30 years then these bonds would from a mortgagor's view appear at least as attractive as traditional callable FRMs.

The paper is organized as follows. We start out by giving a short introduction to delivery option as well as the most common existing mortgage products in Denmark. Then we go through the calculation technique behind the Bolig-X loans, and discuss the valuation technique and their price-yield relationship. With model calculations as benchmark we compare Bolig-X loans with straight ARMs, ARMs with a connected guarantee, as well as callable FRMs. After that we estimate the payments that mortgagors have to pay in order to have a life time cap on the coupon. Finally, we complete the treatment with a discussion. In all calculations it is assumed that mortgagors amortize their debt over 30 year.

2 The Delivery Option

The so-called delivery option is a feature embedded in most Danish mortgages. It allows a mortgagor to buy back his "own" bonds in the market and deliver these bonds to the mortgage credit institution that will cancel his old loan. Basically all mortgages on the Danish mortgage market are pass-through securities, and as a result of this close connection between bonds and loans the delivery option is easily implemented in practice. The delivery option has mostly been

[2]If it is not explictly expressed in the text, the term Bolig-X loan will refer to the type with a cap.

discussed in relation to dynamic debt management strategies, that uses the delivery option to switch to a higher coupon when interest rates have gone up. The idea behind this is to reduce debt and to get higher interest payments which are tax deductable (see e.g. Jakobsen (1992)). Furthermore, the new mortgage provides them with a more valuable option if interest rates are going to decrease again.

However, what many Danish households fail to appreciate is that the delivery option also ensures a much closer match between assets and liabilities. This is particularly important if households are likely to redeem their mortgage for non-financial reasons, as they will have to realize any mismatch between the value of their property and their debt. Hence, the delivery option is more worth to mortgagors, which are more likely to move.

3 Loan types

3.1 Callable Fixed Rate Mortgages (FRM)

Callable FRMs still account for the majority of the mortgage market in Denmark. Typically they have maturities of 10, 20, or 30 year and are callable at par. The bond series are open for issues for a period of three years. Loans can be issued as either cash- or bond loans.

Contrary to the US case mortgagors are not directly presented with a menu of coupons and points. If a mortgagor decides to take out a callable FRM e.g. a callable annuity, with a certain coupon and maturity, the mortgage institution sells a corresponding bond on the stock exchange and just transfers the proceeds to the mortgagor. However, given the combination of amortization profile, coupon, maturity and call option the price of this bond will only by chance trade at par. Furthermore, due to tax-reasons the mortgage credit institutions do not issue bonds trading above par, so the proceeds from a bond sale will generally be lower than the initial principal. This means that a higher principal value is required to obtain a given revenue from the sale. This capital loss works basically in the same way as the coupons and points system in the US.

However, a capital loss can be made tax-deductable in Denmark by issuing a so called cash-loan. On a cash-loan the coupon rate is the yield-to-maturity on the day the bond is issued, which is higher than the coupon on the corresponding bond loan (cash loans are only meaningful when prices are below par). This way the capital loss is transferred into tax-deductable interests and therefore capital gains from redeeming cash-loans are liable to taxation.

3.2 Adjustable Rate Mortgages (ARM)

As interest rates have declined during the last decade Adjustable Rate Mortgages (ARMs) have gained a footing on the Danish mortgage market and now account for a large fraction of new issues. The mortgage credit institution RD's Flexloans (Flexlån) are probably the most well-known ARMs, but similar products are offered by other mortgage credit institutions, including Nykredit, Unikredit, DLR and BRF. Most loans are so called F1-loans, which are funded with 1 year bullet bonds. The advantage of these loans is that the mortgagor

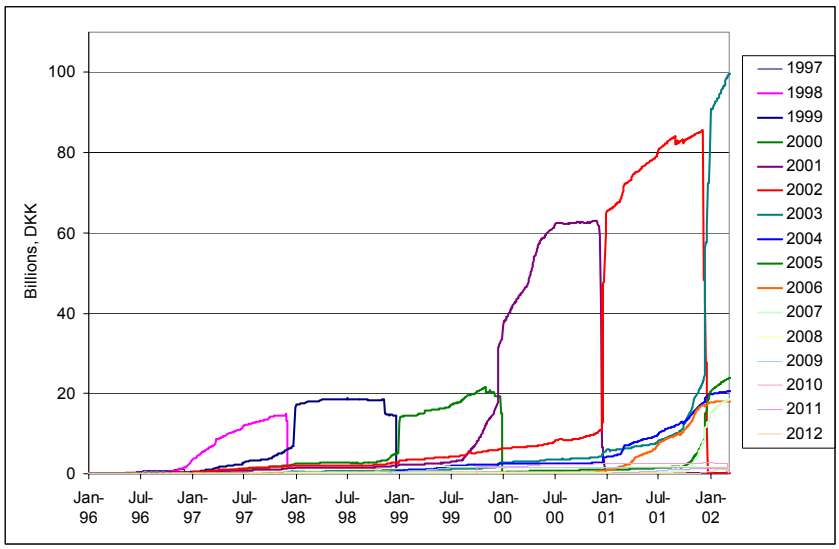

Figure 1: This figure illustrates the nominal outstanding of non-callable bullet bonds used to fund the ARMs in the Danish mortgage market. The figure illustrates how a new 11 year bullet bond is issued when the current 1-year bond matures. The bond series are aggregated based on their maturity indicated in the legend.

refinances his mortgage at the currently low 1 year rate (plus the contribution fee). The disadvantage is that the mortgagor only knows the mortgage payments one year ahead. Figure 1 shows the rapid increase in the nominal amounts of the underlying non-callable bullet bonds sold to finance the ARMs.

These loans are issued as cash loans, but we refer to Tørnes-Hansen (1997) for a thorough introduction.

3.3 Bolig-X mortgages (BXL)

In April 2000 a new adjustable rate product Bolig-X was introduced by Totalkredit. These loans are issued as 5 year bond loans where the coupon is reset twice a year to the 6 Month CIBOR rate[3]. Until now 6 different Bolig-X bond series have been issued, but in this paper we focus on the original three since these are more liquid; 5.156% CIBOR 122c 2005 (abbreviated BX-05 in this paper) is a straight ARM, where the coupon and hence mortgage rate follows the 6 Month CIBOR rate. In the other two bonds 5.356% CIBOR 122c 2005 (BXL-05) and 2007 (BXL-07) the mortgagors pay CIBOR plus a yield spread of 20 basis points (0.2%) for an embedded cap, which guarantees that the coupon rate cannot exceed 7.7% at any time. An overview of the three bond series is given in table 1.

[3]CIBOR rates are published daily by the Danish Central Bank (Danmarks Nationalbank) on the basis of quotes of interbank loan rates from currently 8 Danish banks. Rates of 1, 2, 3, 4, 5, 6, 9 and 12 Months loans are published.

Name	Coupon	Expiry	Price Oct. 1, 2001	Amount DKK mil.
BX-05	5.156%	Jan–01-2005	100.50*	451
BXL-05	5.356%	Jan-01-2005	100.67	5,763
BXL-07	5.356%	Jan-01-2007	100.06	6,702

* Last price Sep 13, 2001

Table 1: Overview Bolig-X loans, October 2001

The Bolig-X loan without cap is a simple way to construct an F1-like mortgage and is very much like the ARMs known from the US mortgage market. At first sight the Bolig-X loan with cap corresponds closely to an F1-loan with an interest rate guarantee as offered by other mortgage credit institutions. However, where the interest rate guaranties offered by the other mortgage credit institutions have failed to take on[4], almost all of Totalkredit's customers have chosen to pay for an embedded cap. This is seen in figure 2.

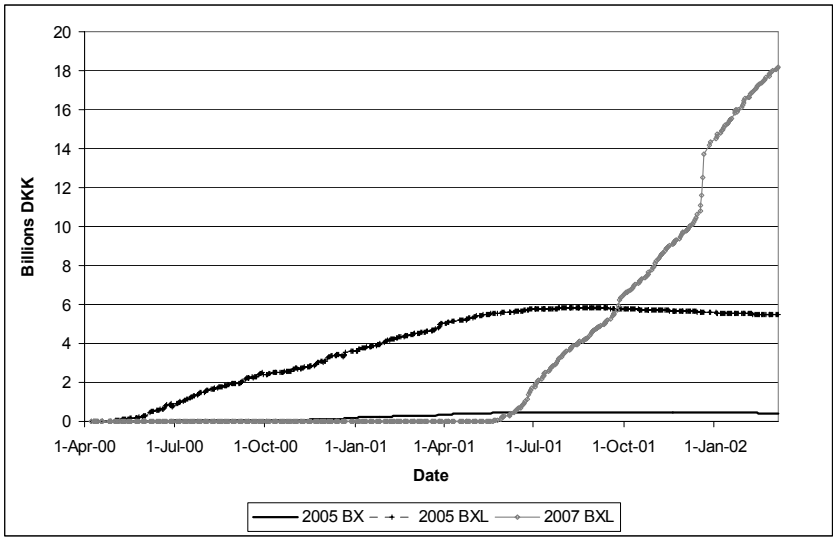

Figure 2: This figure shows the issues in the three bonds series from April 2000 until April 2002. It is obvious that the two series BXL 05 and BLX 07 with the embedded cap are far more popular among mortgagors than the BX 05 with cap.

The success of these loans is likely to be the main reason why Totalkredit has won market shares during the last couple of years.

The Bolig-X bonds have quarterly term dates but the coupon rate is settled twice a year. The coupon rate for the terms October and January is determined as the average of 6 Month CIBOR during the 10 first trade dates in May while the coupon rate for term dates April and July is set during the 10 first trade

[4]To our knowledge no information is published regarding the interest rate guaranties and our assessment is based on conversations with employees at the mortgage institutions.

dates in November. The coupon rate is therefore known approximately 1.5 month before the term period begins, and as such the coupon rate for 1, 2 or 3 terms could be known depending on the trade date.

The mortgagor repays the loan as an annuity with a computational maturity of e.g. 30 years. An example of a possible cash flow is given in table 2.

Term Date	Coupon	Remaining Principal	Interest	Repayment	Payment
Jan, 02	5.36%	996,599	13,400	3,401	16,801
Apr, 02	4.30%	992,429	10,713	4,169	14,883
Jul, 02	4.30%	988,215	10,669	4,214	14,883
Oct, 02	5.80%	984,950	14,329	3,265	17,594
Jan, 03	5.80%	981,638	14,282	3,312	17,594
Apr, 03	7.70%	979,264	18,897	2,374	21,271
Jul, 03	7.70%	976,844	18,851	2,420	21,271
Oct, 03	6.40%	973,725	15,630	3,119	18,748
Jan, 04	6.40%	970,557	15,580	3,169	18,748
Apr, 04	7.70%	967,998	18,683	2,559	21,242
Jul, 04	7.70%	965,390	18,634	2,608	21,242
Oct, 04	7.20%	962,489	17,377	2,901	20,278
Jan, 05	7.20%	0	17,325	962,489	979,814

Table 2: Possible cashflow stream for a Bolig-X loan 2005 with a principal of 1 mill. DKK and a coupon rate cap of 7.7 pct.

With a principal of 1 mil. DKK, a coupon rate of 5.36% and 120 terms to maturity we get a quarterly payment of on term date Jan, 02 of 16,801 DKK. April 2002 the coupon rate is set at 4.3% and with 119 terms left and a remaining principal of 996,599 the payment is 14.883 etc. As is evident a lower coupon results in a higher repayment and vice versa.

April/July 2003 and 2004 illustrate periods where CIBOR plus 20 bp is higher than the cap rate and hence the coupon is set at 7.7%. In January 2005 the bond matures and the mortgagor has to repay the remaining principal of 962.489[5] DKK. In order to make this payment the mortgagor will have to issue a new bond at the prevailing level of interest rates.

The Bolig-X loan can be seen as a package consisting of an adjustable rate bond and a sold cap on 6 month CIBOR with strike 7.7%, and with a stochastic notional following the amortized at the same rate as the underlying bond. A cap is a standardized financial contract which for a given principal pays the difference between the current interest rate and a fixed rate. Prices of caps written on CIBOR are quoted on a regular basis by the major banks and are available on Reuters and Bloomberg terminals. There are, however, problems by using this information directly:

- The cap in the Bolig-X bonds is written on a principal that is reduced in line with the mortgagors repayments. The repayments are unknown in ad-

[5] After some year there could be very different maturities in the BoligX loans as old loans with e.g. 15 years to maturity are to be refinanced in the same bond series as new 30 year mortgages. This will influence the repayment profile and in particular the value of the bonds. We assess that a minimum and maximun maturity within the individual bond series will be necessary.

vance and vary systematically with the development in the interest rates. Increasing interest rates means that the notional of the cap decreases less and hence makes the cap more valuable.

- The cap is written on a 10 day average of CIBOR, which reduces the volatility and hence the caps value relative to a cap written on CIBOR.
- The cap is written on a 6 month rate but with quarterly terms, the strike rate is 7.7% and not equal to the strike rate on the quoted caps, and finally the underlying coupon rate is determined 1.5 month before the term date in contrast to the quoted caps which are settled at CIBOR at the beginning of each term.
- The cap is out of the money and hence less liquid.

These problems make it more difficult to apply simple models to accurately value the cap. For the calculations in this paper we have chosen to use a term structure model which allows us to take these issues into account.

4 Valuation of the mortgage products

4.1 Interest rate model and numerical implementation

The pricing model used in the following analysis is a one-factor Cox, Ingersoll & Ross (1985) term structure model. The short rate r_t dynamics under the risk neutral measure is given by the diffusion

$$dr_t = \kappa(t)(\mu(t) - r_t)\,dt + \sigma(t)\sqrt{r_t}dW_t^Q.$$

In the examples we have chosen to use constant parameters with a current short rate of 3.7% to facilitate replication of the results. The parameters used here was a mean reversion level $\kappa = 20\%$, volatility parameter $\sigma = 0.06$ and the long term level $\mu = 7\%$. These parameters provide a reasonable fit to the Danish swap curve and ATM caps on October 1th, 2001 even though we tend to overvalue the long caps. The choice of the CIR model was made primarily to have some degree of skew in the implied volatilities of caps.

We apply the implementation of a Crank-Nicholson finite-difference solution described in Svenstrup (2001) to solve the fundamental PDE

$$r_t V(t, r_t, A_t) = \frac{\partial V}{\partial t} + \tfrac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial r_t^2} + \mu \frac{\partial V}{\partial r_t} + f(r_t, t)\frac{\partial V}{\partial A_t}, \tag{1}$$

where $V(t, r_t, A_t)$ denotes the time t value of an interest rate dependent claim when the short rate is r_t. A_t denotes an additional state variable used during the fixing periods to capture the path dependency of the running average of the 6 month CIBOR. Notice, as the state variable A_t is updated discretely the last term in the PDE is actually replaced by additional jump conditions.

The implementation takes into account that the coupons are fixed 2-8 months before they are actually paid and hence we only add the present value of the coupons at each fixing date. During the fixing periods we also keep track of the additional state variable representing the accumulated average, but notice that

this is not necessary between fixing periods as the state variable is the 6 Month CIBOR on the first fixing day in each period.

The idea with the chosen model is solely to give a quantitative estimate of the effects of the various input parameters. In practice the model applied would have time dependent parameters in order to obtain a closer fit to the observed term structures of interest rates and volatilities.

4.2 A prepayment model for FRMs

The prepayment model used in the valuation of the FRMs is a version of the ScanRate and Reuters DMBS model based on prepayment data for the period July 1997 to July 2001 (see Pedersen (2000)).

The standard references for a Danish prepayment model are Jakobsen (1992) and Jakobsen (1995). Basically all prepayment models used in the Danish market are variations and minor extensions of the required gain prepayment model developed in Jakobsen (1992). This is a hazard-rate based model for the conditional prepayment rate λ. It stipulates that mortgage holders require a certain net present value gain from prepayment in order to make it worth the effort. Furthermore, the model utilizes information of the borrower composition in a bond series, published by the mortgage credit institutions, to create sub-pools of mortgage holders based on the size of their debt etc.

According to the model the fraction of mortgagors in sub-pool i who prepay at time t can be estimated with

$$\lambda_i(t) = \Phi\left(\boldsymbol{\beta} \cdot f_i\left(x_i; \boldsymbol{\alpha}_i\right)\right),$$

where Φ denotes the standard normal distribution, $\boldsymbol{\beta}$, $\boldsymbol{\alpha}_i$ are paramter vectors, f_i a set of basis functions and x_i a set of covariates. The vector of covariates $\mathbf{x}$ includes the net present vaue gain from prepaying, the time to maturity and a path-dependent burn-out factor. For further details on Danish prepayment models and the valuation of callable FRMs see Jakobsen & Svenstrup (1999), Jakobsen & Rasmussen (1999).

In the valuation of the FRMs we use the additional state-variable in equation (1) to model the path-dependency due to the so-called burn-out effect. Notice that contrary to the US case (see e.g. Schwartz & Torous (1989), Richard & Roll (1989)) we do not include mobility related covariates in the prepayment model.

5 Bolig-X model prices

We now consider the valuation results for the Bolig-X loans. Figure 3 illustrates the model prices on October 1th, 2001 for the three Bolig-X bonds as a function of different levels of the yield curve[6]. Initially the yield premium and cap rate of the bonds are adjusted so as to make the bonds trade close to par. For high interest levels the prices decrease for the two bonds with the cap and the effect is most pronounced for the one with the longest maturity BXL-07.

[6] We have chosen to shift the short rate in the CIR model and maintain the distance between the short rate and its long term level. This will result in almost parallel shifts in the entire yield curve. The first coupon rate January,1th 2002 is fixed at the actual coupon rate of 5.16/5.36%. The subsequent rates vary as determined by the term structure model.

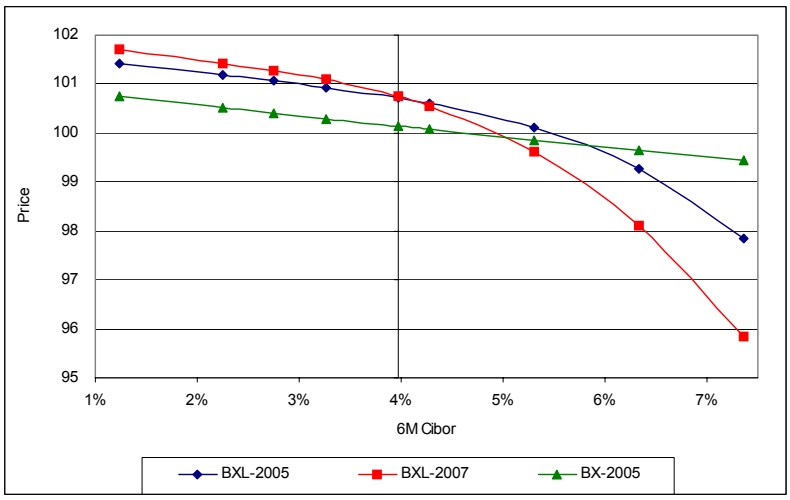

Figure 3: The model prices on October 1th-2001 for Totalkredits Bolig-X bonds at different interest rate levels. The vertical line indicates the initial level of the 6 month CIBOR.

A closer examination shows some interesting effects. On October, 1th the coupon rate for the Jan, 2nd term is fixed at 5.16% for BX-2005. This means that the price is above 100 at low interest rate levels and below for high interest rates. If the interest rates are low the prices of BXL-05/07 with caps are higher than the BX-05 without cap. This is due to a 20 basis points yield premium used to pay for the cap, while the value of the cap is almost zero at low rates. BXL-07 has the yield premium for two years longer than the BXL-05 and hence has a marginally higher price than BXL-05. On the other hand, if the interest rate level is sufficiently high the effect of the cap dominates and the price on BXL-07 is lower than BXL-05. In our calibration of the model to market data on October 1th, 2001, the short rate is 3.7% corresponding to a 6 Month CIBOR on 3.97%. This level of interest rates is marked by the vertical line in the figure. The model gives approximately the same prices for BXL-05 and BXL-07 while the market as shown in table 1 assigns the BXL-05 the highest value.

6 Bolig-X and Adjustable Rate Mortgages

The Bolig-X loan without cap has properties that are very similar to an ARM of the F1 type. As an experiment, let us assume that Totalkredit had chosen to issue BF1 loans without cap where the coupon rate in the beginning of December every year was set equal to the cash loan rate that RD obtains at the auction where they refinance their F1 loans. At the same time assume it is possible for BF1 loans to be sold at price 100. The effect would be that BF1 mortgage holders would have the exact same payment profile before and after tax as the RD mortgage holder with an F1 loan.

On the other hand it will not be certain that BF1 bonds would actually be sold at par. There is a small tax free profit due to appreciation for investors in

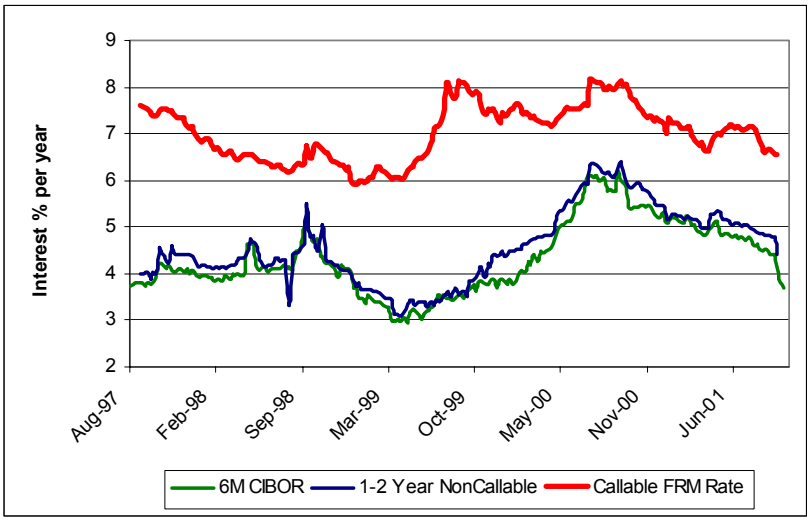

Figure 4: Source: Reuters, The Association of Danish Mortgage Banks

F1 bonds which are typically sold below par. The tax free profit could result in a smaller payments on F1 loans relative to BF1 loans.

In addition the investors would have to assess the liquidity of the BF1 bond relative to an investment where they roll over F1 bonds. A lower liquidity would depress the price on the BF1 bonds.

However, Totalkredit has chosen not to use the F1 coupon but instead 6 month CIBOR as index. As is evident from figure 4 there is a very high correlation between these two rates. On average the 6 month CIBOR is a bit lower than the F1 rate.

CIBOR is defined as a reference rate for loans to a Prime Bank on unsecured basis, while Danish Mortgage Backed Securities are loans backed by property. When buying Bolig-X bonds the investor should hence make an assessment for the remaining maturity of the loan, whether the credit risk of Danish households will be higher or lower than on a CIBOR based loan. With one year ARMs on the other hand investors have the possibility to adjust prices in case of changes in the credit risk.

To summarize, there are differences between plain Bolig-X loans and Flexloans with yearly adjustment. These differences are solely related to differences in transaction costs, tax issues and credit risk and it would be difficult before hand to assess whether Totalkredit's bonds would be priced higher or lower than traditional straight ARMs.

7 Interest rate caps and guaranties

There should be an obvious need to put a ceiling over the payments on an ARM and among others RD and Nykredit offer 1-5 year interest guaranties where the mortgagor can choose a cap rate of e.g. 7% or 8%, see e.g. Thomsen & Tørnes-Hansen (2000) and Bondorf, Sørensen & Carlsen (2000). These interest

rate guaranties are sold as supplementary products to ordinary Danish ARMs.

By selling the interest rate guarantee as a separate product they manage to keep the underlying ARMs simple and liquid and at the same time provide the mortgagors with high flexibility with respect to the design of the guarantee. Despite this, these products have been almost ignored by mortgage holders. In complete contrast the majority of Totalkredit's customers have preferred to pay a yield premium to get a 5 year coupon rate cap.

There are probably several reasons for this. Interest guaranties are sold by a bank attached to the individual mortgage credit institution and until now the guaranties have likely been too expensive relative to the market value of the corresponding option. Furthermore, on early redemption of the mortgage the interest rate guarantee has to be sold back to the bank at a price determinded by the bank, which is probably going to be lower than the market value.

So far the interest rate guarantee has been paid either up-font or through the contribution rate. The first method is not tax-deductable and it is likely that the last method wil be rejected by the tax authorities. Under all circumstances, redemption or transferring of the loan will result in full taxation of any profits on the interest rate guarantee.

In contrast the payment of the guarantee is embedded in the Bolig-X loan as a yield premium on currently 20 basis points. The cap is priced as a part of the bond and hence we expect a relatively sharp pricing. After a severe increase in interest rates the mortgagor could cash in the value of the cap by exercising the delivery option and buy his bonds back in the market. As Bolig-X loans are issued as bond loans any profits are not taxable. The yield premium is, however, fully tax-deductable as well as any transferring of the loan will not release further taxes.

To summarize, we argue that the flexibility which should characterize interest rate guaranties on ARMs is drowned in transaction costs and in the fact that the tax authorities require a close connection between interest rate guarantee and the underlying bond. The Bolig-X construction has from the beginning omitted that flexibility. In return the mortgagor gets the interest rate insurance at the lowest possible price both before and after tax. Last but not least is the marketing. Totalkredit has unambiguously marketed the Bolig-X loan with cap while e.g. RD and Nykredit have hardly marketed their corresponding interest rate guarantee products.

8 Adjustable-Rate Mortgages and callable Fixed-Rate Mortgages

Figure 5 shows the price-yield relationship for the two Bolig-X loans together with a traditional callable fixed-rate annuity 6% 2032 (6-32 denotes coupon and maturity). There are obvious differences. 6-32 is prepayable at price 100 but despite this, the price is getting over 100 because the investors know from experience that many mortgagors are slow to take advantage of their right to prepay. That is investors receive a high coupon for a longer time. Only when all mortgagors decide to prepay the price will be 100. This corresponds to the investor getting the return on his investment reset to the market level.

The coupon rate of the ARM automatically follows changes in the market

rates. Hence for investors this corresponds to a situation where all mortgagors are following an optimal prepayment strategy without transaction costs. The ARM with a 6% cap is therefore less worth to investors than the corresponding callable 6% fixed rate mortgage. On the other hand, mortgagors with ARMs get full pleasure of the decreasing interest rates without paying transaction costs.

In the assessment of the individual mortgagor's interest rate risk it is not an issue that the price of the callable FRM gets over 100, as he or she always are allowed to repay at par. Hence, if we assess a mortgagor's interest rate risk out from holding period costs[7], the two loan will be more or less situated equally.

If the interest rate increases there will be major differences. A short rate on 6% (CIBOR 6.33%) corresponding to a 2.3% increase in interest rate level from October 1th, 2001 will reduce prices of BXL-05 to 99.26 and BXL-07 to 98.10. On the other hand the price of 6-32 will fall to 81.54. A mortgagor who is about to sell his house will therefore be much better off with at traditional callable FRM mortgage in case of an increase in interest rates.

The difference is that the traditional FRM affords protection against increasing interest rates 30 years ahead, while the Bolig-X loans protection expires in 4-6 years at a time where the mortgagor has only repayed a small fraction of the principal. For straight ARMs and Bolig-X loans the difference is of course even larger as the mortgagor is completely unprotected to increasing interest rates.

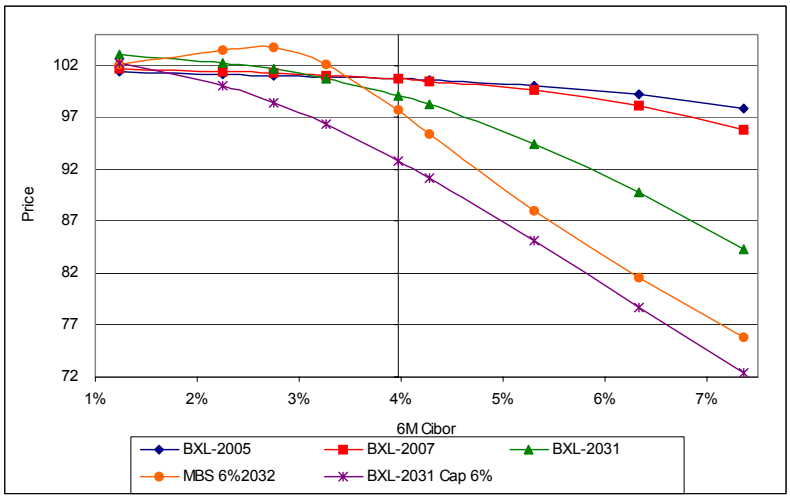

Figure 5: Price yield relationship for Bolig-X loans compared with a traditional 6% Callable Fixed Rate Mortgage.

The interesting part about the Bolig-X loan's construction is that the solution is right ahead. Mortgage credit institutions ought to issue Bolig-X loans with a longer time to maturity e.g. 20-30 years. Hence, the mortgagor would receive protection during the total amortization period, and if the house is to be sold after an interest rate increase the bonds could be redeemed at a price way under 100. This is also illustrated in figure 5 where we have included a hy-

[7]Holding period costs include the accumulated current payments as well as the market value of the remaining principal. See eg. Jakobsen (1998) for definitions and examples.

pothetical BXL-32, that is a 30 year Bolig-X bond with a cap rate of 7.7%. The price yield curve for this bond is much more similar to the traditional callable FRM. For comparison we have furthermore included model prices for a BXL-6%-32 that is a Bolig-X loan with 30 years to maturity and a cap rate of 6%. The price of this bond is always below the price of the corresponding callable loan, but afford the same protection against interest rate changes. At the same time there will be many situations where the mortgagor pays a lower interest. The callable bonds are normally traded with a spread to the swap curve. This spread is included in the price of the 6-32[8]. On the other hand we have not included this spread on the Bolig-X loans. If Bolig-X loans are also traded with a spread their prices will decrease and it would be relatively more expensive to finance property using Bolig-X loans[9]. As discussed in Jakobsen & Svenstrup (2000) the primary reason as to why there is a spread is that the investors require a premium to take on risk about mortgagors prepayment behavior. In a Bolig-X loan there is no uncertainty about future prepayment behavior, so from that point of view we could expect a spread much closer to zero.

9 Market valuation of Bolig-X bonds

Market participants are likely to charge a premium due to the liquidity issues and other non-standard issues in Bolig-X bonds. The 10 day average and the fixing prior to the term period are not similar to the standard fixing rules on the Euro market. To estimate the size of the spread that the mortgages trade with we have set up calculations for all trade dates in the period May 10, 2001 to April 2, 2002. The model has been extended to include time dependent parameters and have been calibrated on a daily basis to the swap curve and a set of ATM swaption quotes. In order account for skew in implied volatilities we would have preferred to include out of- and in the money caps or swaptions in the calibration sample. However, these where not available to us.

In Figure 6 we have plotted the market and model prices for the BXL-07. On the right axis the option adjusted spread has be plotted. The OAS is here defined as the continuously compounded spread that will equate the model and market price when used as an additional discount factor. As we would expect the model prices are higher than the market prices, which gives rise to a positive OAS.

Table 3 shows summary statistics for the net present value (NPV) which is the difference between the model and market price, the OAS and 6 month CIBOR during the period. The median NPV is 1.11 price point and the 25% and 75% percentiles are 0.89 and 1.2 price point. OAS is another way of expressing the NPV, as it is the yield required to amortize the NPV over the remaining life of the bond. Hence, these two measures are highly correlated with a coefficient of 0.97. The OAS has been between 18 and 34 bps during the period and with an average of 25. This seems to be a reasonable spread compared to similar products on the Euro market but maybe on the large side.

[8] The spread to the swap curve usually labeled OAS is about 50 bp at the initial interest rate level at 3.7%, which gives a price of 97.71. The spread decreases to 0 at an interest rate level of 7% in aggrement with the method in Jakobsen & Svenstrup (2000). If the spread is set equal to 0 the price of the 6-32 would be 101.95.

[9] Later in the paper we estimate the spread to be somewhere between 20 and 30 basis

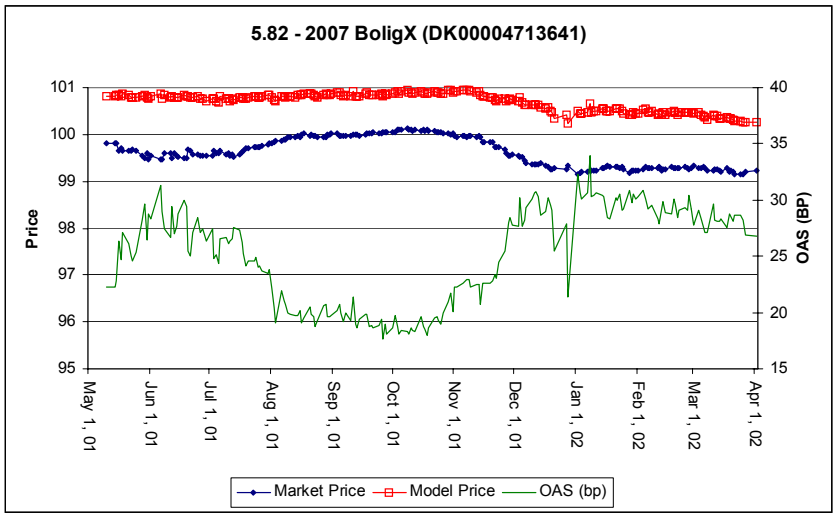

Figure 6: The market and model prices of the 2007 Bolig-X loan with a 7.7% cap rate. The CIR model has been calibrated on a daily basis to the Reuters swap yield curve and a set of swaptions. The option adjusted spread OAS is the additive continuously compounded spread that will equate the model price with the market price.

	Total Obs	**Mean**	**Median**	**Max**	**Min**	**Percentile 25%**	**75%**
NPV	221	1.07	1.11	1.41	0.77	0.89	1.2
OAS (bp)	221	25	26	34	18	21	29
CIBOR 6M %	221	4.13	3.82	5.17	3.48	3.65	4.73

Table 3: Summary statistics for the Net Present Value NPV, the Option Adjusted Spread (OAS) and the 6 month CIBOR during the period May 10th, 2001 to Apr 2nd, 2002.

An ARM settled continuously at the short rate would have a zero duration. However in case of a lower fixing frequency any fixed payments will have a duration corresponding to the time to their maturity, a result well-known from zero coupon bonds. Therefore, the duration of the Bolig-X loans will depend on the number of fixed coupons, which could be from 1 to 3 and the time to the next term date. For the Bolig-X bonds with caps the duration is also effected by the size of the yield premium and the embedded caps. Figure 7 shows the Krone Duration and the Krone Convexity of the BXL 07 bond[10] as well as the 6 Month CIBOR rate. Notice that the embedded cap is out-of-the money as

points.

[10] The Krone Duration and Convexity are here defined as the Krone sensitivity to a discretely compunded parallel shift to the entire yield curve. Both figures are computed from the usual finite difference approximation. Krone or Dollar durations refer to the fact that it is an absolute sensitivity and not a relative sensitivity. To get the relative duration divide by the dirty-price. We use market practice of calculating the duration keeping the OAS fixed.

the CIBOR rate have been less than 5.17% during the period.

Maybe the most interesting observation that can be made from figure 7 is the development of the duration as time passes. In the middle of May 2001 three payments have been fixed with a maturity of roughly 1, 4 and 7 months(this portfolio would approximately have a duration of (1+4+7)/12) 1 year. As time passes the duration of these payments decrease until the beginning of the next fixing period Nov 1, 2001, where only one payment (with maturity 2/12=0.17) is left. Each day of the fixing period a tenth of the next coupon rate is fixed. This is easily seen in the figure, as the duration increases from the beginning to the end of the fixing period. Ignoring the 10 day average during the fixing period would therefore cause durations to be seriously distorted. A study of the importance of index dynamics for the interest rate sensitivity of ARMs can be found in e.g. Stanton & Wallace (1999).

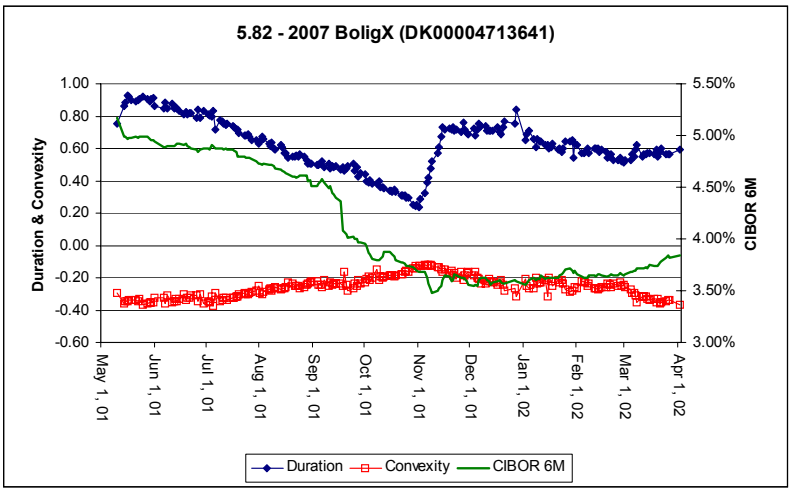

Figure 7: This figure shows the Krone Duration (Dollar Duration) and the Krone Convexity (Dollar Convexity) on the left axis. On the right axis the development in the 6 Month CIBOR is shown.

Tests show that ignoring the 10 day average when fixing future payments in the grid has almost no effect on price (see also Appendix).

10 Design of mortgages

In this section we discuss the design of Bolig-X loans and look at aspects related to counselling of house holders in their mortgage choice. We disregard costs and contribution fees and focus primarily on the payments before tax.

Our main suggestion is to issue 30 year Bolig-X bonds and hence get the interest cap during the entire amortization period. The current Bolig-X bonds and hence the cap matures in 4 to 6 years. In 4 to 6 years the mortgage holder will only have repaid a few percent of the initial principal and as illustrated in figure 8 the value of the remaining principal will almost be par for all interest

rate levels. Even though the Bolig-X loans right now include a better protection than the traditional ARMs, the protection disappears rapidly in a few years.

Worst case scenario is of course an increase in interest rates combined with falling prices of housing. Households facing a borrowing constraint will not be able to pay the mortgage but on the other hand they will not be able to sell their house without loss. It is also too late to refinance into FRMs as the payments in these bonds would be at an even higher level after an increase in interest rates. In this situation compulsory sales would flourish with a further pressure on house prices[11].

On the other hand if the mortgagor had issued an ARM with an interest rate cap, then as long as the household can meet the payment corresponding to the cap rate he or she could stay. Even if the household chooses to sell, the cap, cf. figure 8, will mean that the price of loan will be below 100, so it is likely that the household can repay the debt even with decreasing house prices.

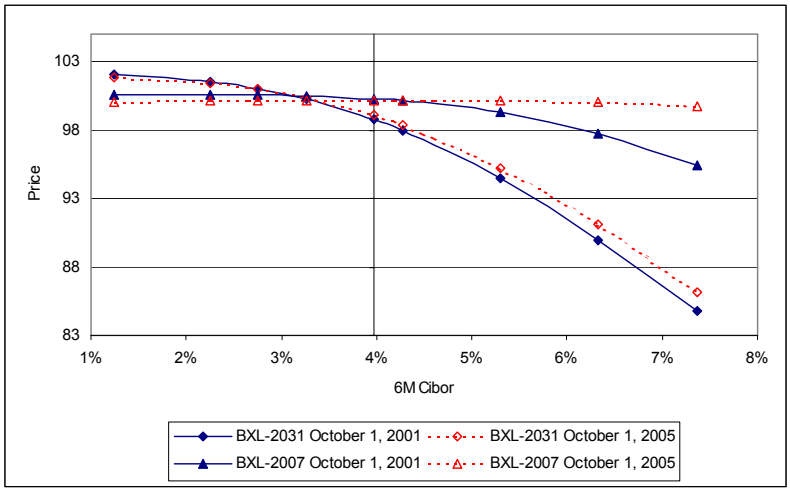

Figure 8: Price movements over time for two Bolig-X bonds. The two full lines show the price yield on October 1, 2001 while the dotted lines show the prices 5 year ahead. This figure illustrate how rapidly the interest rate protection decrease for BLX-07 bond.

Everything equal mortgagors will prefer a long cap on a low level. On the other hand investors will charge for this cap by lower prices and hence higher payments. As it will be evident from below the mortgagors have to weigh low payments right now against high safety.

Figure 9 shows model prices for Bolig-X bonds with a maturity of 5, 10, 15, 20 and 30 years and cap rates on 6, 7, and 8%. The underlying mortgages are all 30 year. In order to compare we also included the price of 30 year Bolig-X loan without cap and a 6% callable FRM computed with and without a option adjusted spread. In all calculations the first coupon has been set equal to the

[11] An alarming feature is that in these days it is argued that house prices are increasing because of the ARMs. We fear that the longer time we have low rates the more reckless the households and their counsellors get and the smaller increase in interest rates is sufficient to ruin the housing market.

level on October 1, 2001, that is 3.97%. The three Bolig-X loans with a cap all include a 20 bp yield premium.

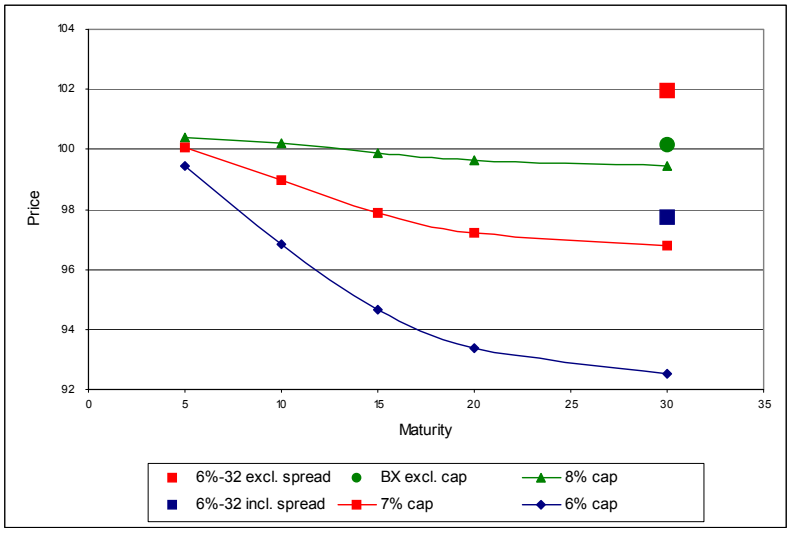

Figure 9: Model prices for bonds depending on cap rate and maturity.

As expected a lower cap rate will result in lower prices e.g. 6% to 8%. The same holds for a longer maturity of the bond and hence a longer cap. Anyway, the difference between a 30 year Bolig-X loan without cap and a Bolig-X with an 8% cap is less than 1 point. If one wants a 6% cap rate it will cost approximately 7.5 price points. In both cases the mortgagor pays an additional 20 bp yield premium. It is interesting that there is less than 1 point difference between getting a 20 and 30 year interest rate cap. This is among other things due to amortization of the principal which is very fast in the last period of the loan. If we compare it to the 30 year 6% callable FRM it will cost 97.72 if we add a spread corresponding to the market spread October 1, 2001, while the price without spread is 101.95. That is, even with the spread the callable FRM is more expensive than a Bolig-X loan with a 6% cap, which reflects the fact that investors receive the full 6% in coupon together with the irrational prepayment behavior in case of decreasing interest rate.

In table 4 and figure 10 we have calculated the actual and maximal payments for the given loans. All loans have a proceed of 1 million DKK. We have disregarded all costs and contributions. For the Bolig-X loans we take a CIBOR rate of 3.97% as of October 1th, 2001 as starting point[12]. If the mortgagor

[12]The mortgage coupon on January 1, 2002 has been set at 5.36% but in the calculations we have chosen to compute the current payment from the interest rate level as of October 1, 2001 as is given by a short rate of 3.7% corresponding to a 6 month CIBOR of 3.974%. We have used 121 quarterly term dates. With a quarterly coupon rate of (3.974+0.2)/4 = 1.04351% and 121 terms the quarterly payment per 1 mil. DKK in principal is 14,590. For a 30 year Bolig-X loan with a 6% cap the price is 92.535. To receive a proceed of 1 mil. DKK the first quarterly payment would be $14{,}590 \cdot 100/92.535 = 15{,}767$, corresponding to 6256 DKK a month. The maximal payment is computed in the same way just with a quarterly coupon of 6%/4=1.5% in stead of 1.04351%. Notice, that in periods with a coupon rate below

	Maturity in Years				
Loan Type	5	10	15	20	30
6% cap	4,891	5,023	5,137	5,209	5,256
7% cap	4,859	4,914	4,967	5,002	5,024
8% cap	4,845	4,854	4,871	4,882	4,889
6% cap Max	6,022	6,185	6,326	6,414	6,471
7% cap Max	6,642	6,717	6,790	6,837	6,868
8% cap Max	7,307	7,321	7,346	7,363	7,374
6-32 incl. spread	-	-	-	-	6,128
6-32 excl. spread	-	-	-	-	5,874
BX no cap	-	-	-	-	4,740

Table 4: Monthly payments before taxes for a mortgage with a proceed of 1 mio. DKK depending on cap rate and maturity of cap. The current payments are only known until next refinancing date. This means that only the 30-year bonds have a life time cap.

chooses a loan with a 6% cap rate she will have to make a monthly payment of 5,256 DKK. If interest rates increase the maximal payment is 6,471 DKK. In case a 8% cap rate is chosen the bonds can be sold at a higher price and the payment right now will be 4,889. In return, the maximal payment before tax is now 7,374 DKK per month. The cheapest mortgage right now is of course the Bolig-X without cap, which corresponds roughly to a standard Danish ARM (F1). Here there is no ceiling over future payments. Finally, there is the traditional callable FRM. This mortgage has the highest mortgage right now but in return this payment can not increase, and is furthermore the mortgage with the lowest maximal payment.

At the moment all historical experiences are disregarded and all mortgagors are advised to issue ARMs. Just a few years ago the advice was that a household should at least afford to finance the house using traditional callable FRMs. If this was the case then maybe an ARM could be considered. Even this piece of advice is problematic as the ARM could show to be more expensive than the alternative FRM that originally was rejected.

If the market offers a range of loans with lifetime cap rates the counselling of mortgage holders would be much concrete. A mortgagor could use the maximal possible payment as reference point. Given this maximal payment there will be a tradeoff between proceeds and the initial payment, such that a higher proceed will require a less risky mortgage. This is completely in line with the US litterature discussed in the introduction.

If we focus on the payments before taxes there is a relatively small difference between mortgages with a 6% and 8% cap rate. The difference is larger in an after tax consideration as the loan with a 6% cap is issued as a bond loan at price 92.5, that is with a loss due to depreciation that is not tax-deductable. In practice one would either issue the bonds as cash loans or issue bonds with a higher cap rate e.g. 7% or 8%[13].

the maximum rate the debt is repayed faster, which subsequent would reduce the maximal payment.

[13]It is also possible to increase the yield premium to e.g. 60 bp. This would increase the price from 92.53 to 94.27. The problem is that an increased yield premium does not have an

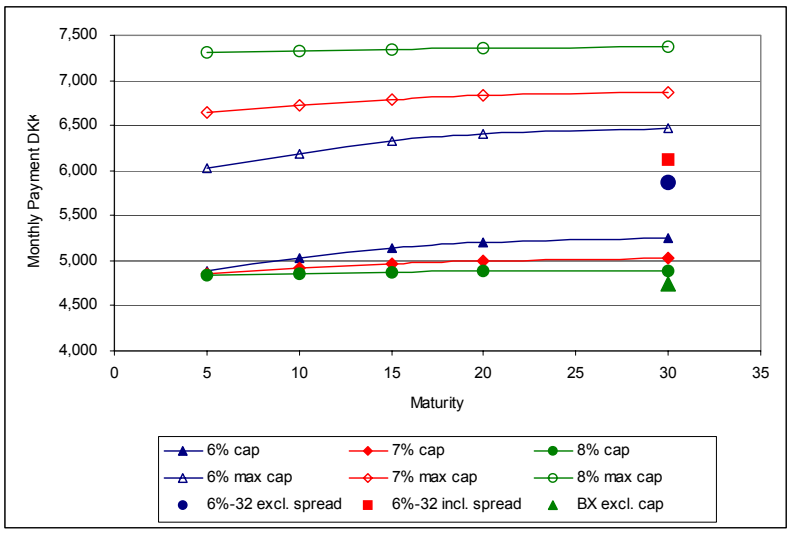

Figure 10: Actual and maximal payments before tax as a function of maturity for loans with a proceed equal to 1 million DKK. Series with full marks represent the current payment and the dotted marks corresponding maximal possible payments.

11 Discussion

In our opinion long ARMs with life time caps have the potential to create a seminal innovation of the Danish mortgage market. In contrast to the existing straight ARMs these bonds put a necessary ceiling on the mortgage payments. Allowing ordinary households to buy houses on the borderline of their financial capabilities and only financing the first year of a 30 year amortization period is in our opinion totally irresponsible. With a life time cap the maximal payment is guaranteed and the calculations show that the payments will only be marginally higher than the unsecured loan. The risk has not disappeared but it has been transferred against charge from the mortgagor to the financial investors in the money- and bond markets. Furthermore, the existence of the cap increase the value of the delivery option.

Compared to traditional Danish ARMs with interest rate guaranties the Bolig-X product is simpler, more easily clarified in relation to taxes[14] and probably also cheaper. At least they sell better.

In relation to the traditional callable FRMs the Bolig-X bonds give similar

effect in those scenarios where the cap already is binding, so that a very large yield premium is required in order increase the price.

On the other hand as the yield premium works as a fixed payment during the entire maturity of the bond it increases the mortgagors minimum payment and hence the price risk when interest rates decrease.

The maximal price is obtained at a premium of 6%, that is the mortgagor will never pay less than 6%. But that is exactly a 30 year non callable loan - which is something no household wants to issue.

[14] There has been issues with tax authorities but according to Totalkredit these issues have been settled.

insurance when interest rates increase and automatic payment reduction when they fall. Mortgagors avoid transaction costs when prepaying and investors are able to cover their interest rate risk using standard interest rate products without worrying about modelling prepayment behavior and other difficulties related to the traditional FRMs.

Just as for callable FRMs there is a risk of illiquid bond series. If the level of interest rates gets close to or higher than the cap rate the prices will drop too far below 100 and hence prevent new issues in that series. In case of a lower interest rate, mortgagors are likely to prefer series closer to the money. However, old series could be reopened if interest rates return to the initial level. In contrast to existing callable FRM series these are not required to be closed for new issues when prices exceed 100.

Illiquid bonds at prices below 100 could mean that mortgagors have to buy at a premium in order to redeem their loans. This is also the situation for traditional callable FRMs. With an adjustable rate it is unlikely that mortgagors risk redeeming their bonds at prices above 100. Under all circumstances this type of bonds will not depend upon the behavior of the borrowers and the individual mortgage institutions series will be close substitutes.

In the sketched bond type the interest rate cap cannot be changed. In practice it could happen by a simple renewal of the loans. After an increase in interest rates the mortgagor could redeem his old loan at market price and issue a new bond with a higher cap rate. After an interest rate decrease the mortgagor could against an additional payment take out a new loan with lower cap rate.

As an alternative to these prepayments the institutions could choose to embed an automatically adjustment in the loan. In the US market ARMs are issued with so called rolling caps as well as lifetime caps.

A traditional problem with adjustable rate mortgages is to find a stable index rate. It won't be a problem that Totalkredit's bonds with a nominal of 25 billion DKK is linked to 6 month CIBOR, but if the mortgage credit institutions are to fix 3-400 billion DKK on CIBOR in too short periods every year, then a tremendous focus would be put on the banks that report CIBOR. Similarly the liquidity in traditional F1 ARMs would probably drop dramatically if RD offered loans indexed after the F1-rate but also had 10, 20 or 30 year caps.

The starting point for our calculations is that long Bolig-X like bonds are priced in line with or better than traditional callable FRMs. It is naturally an open question whether the market is willing to absorb large issues of Bolig-X like series even though a corresponding amount of the traditional bonds are repaid.

All these considerations are of course of hypothetical nature and the proposal to Totalkredit and other mortgage credit institutions is simple to try. The product could die quietly but it could also turn the Danish mortgage market up and down. Apparently there are lots of mortgagors who dare to refinance their house once a year in order to obtain a lower payment right now, but it is our assessment that even more are willing to pay a bit extra if they can budget with a maximal mortgage payment the next 30 year.

12 Conclusion

In the Danish mortgage market mostly straight ARMs have been issued. However, the US mortgage choice literature indicates that hybrid ARMs are attractive to mortgagors and the experiences from the Bolig-X bonds support this. Furthermore, our calculations indicate that the limited effect on monthly payments of buying life-time out-the-money caps, will be of interest for many of the mortgagors currently rolling over their 30 year mortgages at the 1 year rate. It appears very reasonable that risk averse mortgagors will be willing to insure themselves against worst case losses. The more likely it is that interest rates are going to increase, the more expensive it will be to get a life time ceiling over the mortgage payment. However, this is not a reason to issue a straight ARM - on the contrary.

Also of interest to e.g. the US mortgage market, we argue that the delivery option is a very important and efficient means to ensure a tighter match between the assets and liabilities in a household portfolio. Furthermore, a by-product of the delivery option is an increase in the mobility of the labour force as a whole, which also could be of macro-economic importance.

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A Appendix

Tests show that ignoring the 10 day average when fixing future payments in the grid has almost no effect. The length of the fixing period is simply to short relative to the time between two fixings in order to have a significant impact of the index volatility. To illustrate this consider for example the variance of an average in a Vasicek model

$$dr_t = \kappa(\mu - r_t)\,dt + \sigma dW_t.$$

If we let $A_T = \frac{1}{n}\sum_{i=1}^{n} r_{t_i}$ it is easily seen using the property of independent increments that

$$\begin{aligned}
Var(A_T) &= Var\left(\frac{1}{n}\sum_{i=1}^{n} r_{t_i}\right) = \frac{1}{n^2} Var\left(\sum_{i=1}^{n} r_{t_i}\right) \\
&= \frac{1}{n^2} Var\left(n r_{t_1} + (n-1)(r_{t_2} - r_{t_1}) + .. + 1\left(r_{t_n} - r_{t_{n-1}}\right)\right) \\
&= \frac{1}{n^2} Var\left(\sum_{i=1}^{n} (n-i+1)\left(r_{t_i} - r_{t_{i-1}}\right)\right) \\
&= \sum_{i=1}^{n} \frac{(n-i+1)^2}{n^2} Var\left(r_{t_i} - r_{t_{i-1}}\right) \\
&= \sigma^2 \sum_{i=1}^{n} \frac{(n-i+1)^2}{n^2}(t_i - t_{i-1}) \\
&= \sigma^2 \sum_{i=1}^{n} \frac{(n-i+1)^2}{n^2}\Delta_i.
\end{aligned}$$

Hence if the Δ_1 is large relative to the rest of the intervals the decrease in variance is small. In the case of the Bolig-X loans even for the first fixing there will be 6 months and the fixing period is 10 days then $\Delta_1 = 0.5$ and $\Delta_i = 1/252$ hence the period from t_0 to t_1 contributes with 98% of the total variance of A_T. For following fixing periods this is even more pronounced as Δ_1 is even larger while other Δ_i's remain the same. Of course this argument does not hold as we approach a fixing period, but then the value of the cap is not as sensitive to the volatility.